Summercroft Primary School Calculation Policy

Introduction

At Summercroft we believe that children should be introduced to the processes of calculation through practical, oral and mental activities.

The overall aim is that when children leave Summercroft they:

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using appropriate numbers;
- make use of diagrams and informal notes to help record steps;
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- have developed problem solving skills when using and applying mathematics in written and practical activities;
- use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

Mental methods of calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another. Ongoing oral and mental work provides practice and consolidation of these ideas. The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined.

Secure mental calculation requires the ability to:

- recall key number facts instantly – for example, all addition and subtraction facts for each number to at least 10 (Year 2), sums and differences of multiples of 10 (Year 3) and multiplication facts up to 10 × 10 (Year 4);
- use taught strategies to work out the calculation – for example, recognise that addition can be done in any order and use this to add mentally a one-digit number or a multiple of 10 to a one-digit or two-digit number (Year 1), partition two-digit numbers in different ways including into multiples of ten and one and add the tens and units separately and then recombine (Year 2), when applying mental methods in special cases (Year 5);
- understand how the rules and laws of arithmetic are used and applied – for example, to add or subtract mentally combinations of one-digit and two-digit numbers (Year 3), and to calculate mentally with whole numbers and decimals (Year 6).

Written methods of calculation

The aim is that by the end of Key Stage 2, the great majority of children should be able to use an efficient written method for each operation with confidence and understanding. This guidance promotes the use of what are commonly known as 'standard' written methods – methods that are efficient and work for any calculations, including those that involve whole numbers or decimals. They are compact and consequently help children to keep track of their recorded steps. Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator. We want children to know that they have such a reliable, written method to which they can turn when the need arises.

In setting out these aims, the intention is that we adopt greater consistency in our approach to calculation. The challenge is for our teachers is determining when their children should move on to a refinement in the method and become confident and more efficient at written calculation.

Children should be equipped to decide when it is best to use a mental, written or calculator method based on the knowledge that they are in control of this choice as they are able to carry out all three methods with confidence.

The correct use of vocabulary and mathematical symbols will be reinforced to ensure that it is mathematically correct.

Choosing the appropriate strategy

Children need to be confident in deciding when a mental or written strategy is appropriate. For this reason teachers need to ensure that pupils have the opportunity to make appropriate decisions.

Objectives

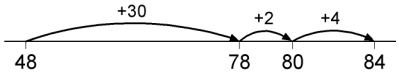
The objectives in the revised Framework show the progression in children's use of written methods of calculation in the strands 'Using and applying mathematics' and 'Calculating'.

Using and applying mathematics	Calculating
Foundation stage • Take part in mathematical rhymes, songs and games. • Solve problems involving counting, adding, subtracting, in the context of numbers, measures or money, for example recognising differences in quantities when counting 2 sets of objects, comparing sizes of 2 objects.	• Respond to the vocabulary involved in addition and subtraction. • Recognise differences in quantities. • Find 1 more or less from a group of up to 5 (then 10) objects. • Relate addition to combining 2 groups. • Relates subtraction to taking away. • Use a range of strategies for addition and subtraction, including mental recall of number.
Year 1 •Solve problems involving counting, adding, subtracting, doubling or halving in the context of numbers, measures or money, for example to 'pay' and 'give change' •Describe a puzzle or problem using numbers, practical materials and diagrams; use these to solve the problem and set the solution in the original context	•Relate addition to counting on; recognise that addition can be done in any order; use practical and informal written methods to support the addition of a one-digit number or a multiple of 10 to a one-digit or two-digit number •Understand subtraction as 'take away' and find a 'difference' by counting up; use practical and informal written methods to support the subtraction of a one-digit number from a one-digit or two-digit number and a multiple of 10 from a two-digit number •Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences
Year 2 •Solve problems involving addition, subtraction, multiplication or division in contexts of numbers, measures or pounds and pence •Identify and record the information or calculation needed to solve a puzzle or problem; carry out the steps or calculations and check the solution in the context of the problem	•Represent repeated addition and arrays as multiplication, and sharing and repeated subtraction (grouping) as division; use practical and informal written methods and related vocabulary to support multiplication and division, including calculations with remainders •Use the symbols +, –, ×, ÷ and = to record and interpret number sentences involving all four operations; calculate the value of an unknown in a number sentence (e.g. $\square \div 2 = 6$, $30 - \square = 24$)
Year 3 •Solve one-step and two-step problems involving numbers, money or measures, including time, choosing and carrying out appropriate calculations •Represent the information in a puzzle or problem using numbers, images or diagrams; use these to find a solution and present it in context, where appropriate using £.p notation or units of measure	•Develop and use written methods to record, support or explain addition and subtraction of two-digit and three-digit numbers •Use practical and informal written methods to multiply and divide two-digit numbers (e.g. 13×3, $50 \div 4$); round remainders up or down, depending on the context •Understand that division is the inverse of multiplication and vice versa; use this to derive and record related multiplication and division number sentences
Year 4 •Solve one-step and two-step problems involving numbers, money or measures, including time; choose and carry out appropriate calculations, using calculator methods where appropriate •Represent a puzzle or problem using number sentences, statements or diagrams; use these to solve the problem; present and interpret the solution in the context of the problem	•Refine and use efficient written methods to add and subtract two-digit and three-digit whole numbers and £.p •Develop and use written methods to record, support and explain multiplication and division of two-digit numbers by a one-digit number, including division with remainders (e.g. 15×9, $98 \div 6$)
Year 5 •Solve one-step and two-step problems involving whole numbers and decimals and all four operations, choosing and using appropriate calculation strategies, including calculator use •Represent a puzzle or problem by identifying and recording the information or calculations needed to solve it; find possible solutions and confirm them in the context of the problem	•Use efficient written methods to add and subtract whole numbers and decimals with up to two places •Use understanding of place value to multiply and divide whole numbers and decimals by 10, 100 or 1000 •Refine and use efficient written methods to multiply and divide HTU × U, TU × TU, U.t × U and HTU ÷ U
Year 6 •Solve multi-step problems, and problems involving fractions, decimals and percentages; choose and use appropriate calculation strategies at each stage, including calculator use •Represent and interpret sequences, patterns and relationships involving numbers and shapes; suggest and test hypotheses; construct and use simple expressions and formulae in words then symbols (e.g. the cost of *c* pens at 15 pence each is 15*c* pence)	•Use efficient written methods to add and subtract integers and decimals, to multiply and divide integers and decimals by a one-digit integer, and to multiply two-digit and three-digit integers by a two-digit integer

Mental methods for addition of whole numbers

Foundation	Say and use the number names in order in familiar settings Recite the number names in order, continuing the count forward from a given number. Count reliably up to 10 everyday objects Count in tens and twos Find one more than a number from 1 to 10 Begin to relate addition to combining two groups of objects Partition a given number of objects into two groups Select two groups of objects to make a given total.
Year 1	Count on in ones and tens from any small number. Begin to know what each digit in a two-digit number represents. Partition a 'teens' number and begin to partition larger two-digit numbers. Within the range 0 to 30, say the number that is 1 or 10 more or less than any given number. Know by heart all pairs of numbers with a total of 10. Begin to know addition facts for all pairs of numbers with a total of at least 10. Use known number facts and place value to add a pair of numbers mentally within the range 0 to at least 10, then 0 to at least 20.
Year 2	Count on in ones or tens, starting from any two-digit number. Know what each digit in a two-digit-number represents, and partition two digit numbers into a multiple of tens and ones. Say the number that is 1 or 10 more than any given two-digit number. Know by heart all addition facts for each number to at least 10, all pairs of numbers with a total of 20, all pairs of multiples with a total of 100. Find a small difference by counting up from the smaller to the larger number. Add 9/11 by adding 10 and adjusting by 1. Begin to add 19/21. Use known number facts and place value to add mentally.
Year 3	Count on in tens or hundreds, starting from any two- or three-digit number. Know what each digit represents, and partition three-digit numbers into a multiple of 100, a multiple of ten and ones. Say the number that is 1, 10 or 100 more or less than any two- or three-digit number. Know by heart all addition facts for each number to 20 and all pairs of multiples of 100 with a total of 1000. Derive quickly all pairs of multiples of 5 with a total of 100. Add mentally a near multiple of 10 to or from a two-digit number, by adding a multiple of ten then adjusting.
Year 4	Partition numbers into thousands, hundreds, tens and ones. Add 1, 10, 100 or 1000 to any integer and count on in tens, hundreds or thousands from any whole number up to 10 000. Consolidate knowing by heart addition facts for all numbers to 20. Derive quickly all number pairs that total 100, all pairs of multiples of 50 with a total of 1000. Count on repeated steps of 1, 10 or 100. Add the nearest multiple of 10 then adjust. Use known number facts and place value to add mentally.
Year 5	Know what each digit represents in a number with up to two-decimal places. Derive quickly decimals that total 1, all two-digit pairs that total 100, all pairs of multiples of 50 with a total of 1000. Add the nearest multiple of 10 or 100 then adjust. Use known number facts or place value for mental addition.
Year 6	Know what each digit represents in a number with up to three decimal places. Add the nearest multiple of 10, 100 or 1000, then adjust. Use known number facts and place value to consolidate mental addition.

Written methods for addition of whole numbers

Phase 1: Simple number sentences, using number lines.	
Phase 2: The empty number line • The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and units separately, often starting with the tens. Children need to be able to partition numbers in ways other than into tens and units to help them make multiples of ten by adding in steps. • The empty number line helps to record the steps on the way to calculating the total	Phase 2 Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10. 8 + 7 = 15  48 + 36 = 84  This will also include the use of the 100 square to reinforce the use of partitioning.
Phase 3: Expanded method in columns • Move on to a layout showing the addition of the tens to the tens and the units to the units separately • The addition of the tens in the calculation 47 + 76 is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'. • The expanded method leads children to the more compact method so that they understand its structure and efficiency.	Phase 3 Write the numbers in columns. Adding the units first: 47 + 76 13 110 123
Phase 4: Column method • In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'. • Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.	Phase 4 47 + 76 = 123 (1 1) 258 + 87 = 345 (1 1) 366 + 458 = 824 (1 1) Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.
Phase 5: Column method expanded to decimals **In this phase the standard method is expanded to include the use of decimals.**	3 2 4 . 6 5 4 6 . 6 8 7 1 . 2 1 1

Mental methods for subtraction of whole numbers

Foundation	Recite the number names in order, continuing the count forward or backwards from a given number. Begin to relate subtraction to 'taking away'. Remove a smaller number from a larger number and find how many are left by counting back. Begin to find out how many have been removed from a larger group of objects by counting up from a number. Work out by counting how many more are needed to make a larger number.
Year 1	Count on or back in ones from any small number, and in tens from and back to zero. Begin to know what each digit in a two-digit number represents. Partition a 'teens' number and begin to partition larger two-digit numbers. Within the range 0 to 30, say the number that is 1 or 10 more or less than any given number. Know by heart all pairs of numbers with a total of 10. Begin to know addition facts for all pairs of numbers with a total of at least 10, and the corresponding subtraction facts. Use known number facts and place value to add or subtract a pair of numbers mentally within the range 0 to at least 10, then 0 to at least 20.
Year 2	Count on or back in ones or tens, starting from any two-digit number. Know what each digit in a two-digit-number represents, and partition two digit numbers into a multiple of tens and ones. Say the number that is 1 or 10 more or less than any given two-digit number. Know by heart all addition and subtraction facts for each number to at least 10, all pairs of numbers with a total of 20, all pairs of multiples with a total of 100. Find a small difference by counting up from the smaller to the larger number. Add/subtract 9/11 by adding/subtracting 10 and adjusting by 1. Begin to add/subtract 19/21. Use known number facts and place value to add/subtract mentally.
Year 3	Count on or back in tens or hundreds, starting from any two- or three-digit number. Know what each digit represents, and partition three-digit numbers into a multiple of 100, a multiple of ten and ones. Say the number that is 1, 10 or 100 more or less than any two- or three-digit number. Know by heart all addition and subtraction facts for each number to 20 and all pairs of multiples of 100 with a total of 1000. Derive quickly all pairs of multiples of 5 with a total of 100. Find a small difference by counting up from the smaller to the larger number. Add and subtract mentally a near multiple of 10 to or from a two-digit number, by adding or subtracting a multiple of ten then adjusting.
Year 4	Partition numbers into thousands, hundreds, tens and ones. Add/subtract 1, 10, 100 or 1000 to/from any integer and count on or back in tens, hundreds or thousands from any whole number up to 10 000. Consolidate knowing by heart addition and subtraction facts for all numbers to 20. Derive quickly all number pairs that total 100, all pairs of multiples of 50 with a total of 1000. Find a small difference by counting up. Count on or back in repeated steps of 1, 10 or 100. Add or subtract the nearest multiple of 10 then adjust. Use known number facts and place value to add or subtract mentally.
Year 5	Know what each digit represents in a number with up to two-decimal places. Derive quickly decimals that total 1, all two-digit pairs that total 100, all pairs of multiples of 50 with a total of 1000. Find differences by counting up through next multiple of 10, 100 or 1000. Add/subtract the nearest multiple of 10 or 100 then adjust. Use known number facts or place value for mental addition and subtraction.
Year 6	Know what each digit represents in a number with up to three decimal places. Consolidate finding a difference by counting on. Add/subtract the nearest multiple of 10, 100 or 1000, then adjust. Use known number facts and place value to consolidate mental addition/subtraction.

Written methods for subtraction of whole numbers

Phase 1: Simple number sentences counting back on number lines.

Phase 2: Using the empty number line

- The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten.
- The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47.
- With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 – 12, 86 – 77 or 43 – 28.

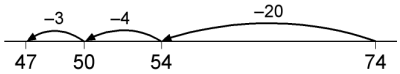
The notes below give more detail on the counting-up method using an empty number line.

Phase 2

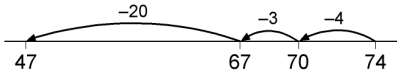
Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

15 – 7 = 8

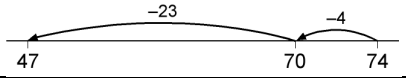
74 – 27 = 47 worked by counting back:



The steps may be recorded in a different order:



or combined:



Phase 3: Expanded layout, leading to column method

- Partitioning the numbers into tens and units and writing one under the other mirrors the column method, where units are placed under units and tens under tens.
- This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills.
- The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.

Phase 3

Partitioned numbers are then written under one another:

Example: 74 – 27

$$\begin{array}{r} 70+4 \\ -\ 20+7 \\ \hline \end{array} \qquad \begin{array}{r} \overset{60}{\cancel{70}}+\overset{14}{\cancel{4}} \\ -\ 20+7 \\ \hline 40+7 \end{array} \qquad \begin{array}{r} \overset{6}{\cancel{7}}\ \overset{14}{\cancel{4}} \\ -\ 2\ 7 \\ \hline 4\ 7 \end{array}$$

Example: 741 – 367

$$\begin{array}{r} 700+40+1 \\ -\ 300+60+7 \\ \hline \end{array} \qquad \begin{array}{r} \overset{600}{\cancel{700}}+\overset{130}{\cancel{40}}+\overset{11}{\cancel{1}} \\ -\ 300+60+7 \\ \hline 300+70+4 \end{array} \qquad \begin{array}{r} \overset{6}{\cancel{7}}\ \overset{13}{\cancel{4}}\ \overset{11}{\cancel{1}} \\ -\ 3\ 6\ 7 \\ \hline 3\ 7\ 4 \end{array}$$

The expanded method for three-digit numbers

Example: 563 – 241, no adjustment or decomposition needed

Expanded method	leading to
500 + 60 + 3	563
– 200 + 40 + 1	– 241
300 + 20 + 2	322

Start by subtracting the units, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take away forty', not 'six take away four'.

Example: 563 – 271, adjustment from the hundreds to the tens, or partitioning the hundreds

$$\begin{array}{r} 500+60+3 \\ -\ 200+70+1 \\ \hline \end{array} \qquad \begin{array}{r} 400+160+3 \\ -\ 200+\ 70+1 \\ \hline 200+\ 90+2 \end{array} \qquad \begin{array}{r} \overset{400}{\cancel{500}}+\overset{160}{\cancel{60}}+3 \\ -\ 200+70+1 \\ \hline 200+90+2 \end{array} \qquad \begin{array}{r} \overset{4}{\cancel{5}}\ \overset{16}{\cancel{6}}3 \\ -\ 2\ 7\ 1 \\ \hline 2\ 9\ 2 \end{array}$$

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and units components of the number, and how 500 + 60 can be partitioned into 400 + 160. The subtraction of the tens becomes '160 minus 70', an application of subtraction of multiples of ten.

Example: 503 – 278, dealing with zeros when adjusting

$$\begin{array}{r} 500+\ 0+3 \\ -\ 200+70+8 \\ \hline \end{array} \qquad \begin{array}{r} 400+90+13 \\ -\ 200+70+\ 8 \\ \hline 200+20+\ 5 \end{array} \qquad \begin{array}{r} \overset{400}{\cancel{500}}+\overset{90}{\cancel{0}}+\overset{13}{\cancel{3}} \\ -\ 200+70+8 \\ \hline 200+20+5 \end{array} \qquad \begin{array}{r} \overset{4}{\cancel{5}}\ \overset{9}{\cancel{0}}\ \overset{13}{\cancel{3}} \\ -\ 2\ 7\ 8 \\ \hline 2\ 2\ 5 \end{array}$$

Here 0 acts as a place holder for the tens. The adjustment has to be done in two stages. First the 500 + 0 is partitioned into 400 + 100 and then the 100 + 3 is partitioned into 90 + 13.

Mental methods for multiplication of whole numbers

Foundation	Count in tens. Count in twos.
Year 1	Count on and back in tens from and back to zero. Count on in twos from zero. Count in steps of five from zero. Begin to count in steps of three from zero. Begin to know what each digit in a two-digit number represents. Partition a 'teens' number and begin to partition larger two-digit numbers into a multiple of ten and ones.
Year 2	Count in hundreds from and back to zero. Count on in twos from and back to zero. Count in steps of 3, 4 or 5 to at least 30, from and back to zero. Know what each digit in a two-digit number represents, and partition two-digit numbers into a multiple of ten and ones. Understand the operation of multiplication as repeated addition or as describing an array. Know by heart multiplication facts for the 2, 5 and 10 times-tables. Use known number facts and place value to carry out mentally simple multiplications. Derive quickly doubles of all multiples of numbers to at least 15 and multiples of 5 to 50.
Year 3	Count on in steps of 3, 4 or 5 from any small number to at least 50 and then back again. Know what each digit represents, and partition three-digit numbers into a multiple of 100, a multiple of ten and ones. Know by heart multiplication facts for the 2, 3, 4, 5 and 10 times-tables. To multiply by 10/100, shift the digits one/two places to the left. Use known number facts and place value to carry out mentally simple multiplications.
Year 4	Partition numbers into thousands, hundreds, tens and ones. Multiply and divide any integer up to 1000 by 10 and 100 and understand the effect. Know by heart multiplication facts for 2, 3, 4, 5, 6, 9 and 10 times-tables. Begin to know 7 and 8 times tables. Partition (e.g. 23 x 4 = (20 x 4) + (3 x 4)). Use known facts and place value to multiply integers, including by 10 and then 100.
Year 5	Know what each digit represents in a number with up to two decimal places. Multiply and divide any positive integer up to 10 000 by 10 or 100 and understand the effect. Know by heart all multiplication facts up to 10 x 10. Partition (e.g. 47 x 6 = (40 x 6) + (7 x 6)). Use known facts and place value to multiply mentally.
Year 6	Multiply and divide decimals mentally by 10 or 100, and integers by 1000, and explain the effect. Consolidate knowing by heart multiplication facts up to 10 x 10 Partition (e.g. 87 x 6 = (80 x 6) + (7 x 6)). Use known number facts and place value to consolidate mental multiplication.

Written methods for Multiplication of whole numbers

<table>
<tr><td>Phase 1: Number sentences recorded horizontally.

Making arrays.</td><td></td></tr>
<tr><td>Phase 2: Mental multiplication using partitioning
• Mental methods for multiplying TU × U can be based on the distributive law of multiplication over addition. This allows the tens and units to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the units can be multiplied first but it is more common to start with the tens.</td><td>Phase 2

Informal recording in Year 4 might be:

43
40 + 3
↓ ↓ ×6
240 + 18 = 258

Also record mental multiplication using partitioning:
$14 \times 3 = (10 + 4) \times 3$
$= (10 \times 3) + (4 \times 3) = 30 + 12 = 42$
$43 \times 6 = (40 + 3) \times 6$
$= (40 \times 6) + (3 \times 6) = 240 + 18 = 258$</td></tr>
<tr><td>Phase 3: The grid method
• As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.
• It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.</td><td>Phase 3

38 × 7 = (30 × 7) + (8 × 7) = 210 + 56 = 266
<table><tr><td>×</td><td>7</td></tr><tr><td>30</td><td>210</td></tr><tr><td>8</td><td>56</td></tr><tr><td></td><td>266</td></tr></table></td></tr>
<tr><td>Phase 4: Expanded short multiplication
• The next step is to represent the method of recording in a column format, but showing the working.
• Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is ‘thirty multiplied by seven’, not ‘three times seven’, although the relationship 3 × 7 should be stressed.
• Most children should be able to use this expanded method for TU × U by the end of Year 4.</td><td>Phase 4

3 8
7 x
5 6
2 1 0
2 6 6</td></tr>
<tr><td>Phase 5: Short multiplication
• The recording is reduced further, with carry digits recorded below the line.
• If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 3.</td><td>Phase 5

38
× 7
266
5

The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.</td></tr>
<tr><td>Phase 6: Two-digit by two-digit products
• Extend to TU × TU, asking children to estimate first.
• Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product.
• As in the grid method for TU × U in stage 4, the first column can become an extra top row as a stepping stone to the method below.</td><td>Phase 6

56 × 27 is approximately 60 × 30 = 1800.
<table><tr><td>×</td><td>20</td><td>7</td><td></td></tr><tr><td>50</td><td>1000</td><td>350</td><td>1350</td></tr><tr><td>6</td><td>120</td><td>42</td><td>162</td></tr><tr><td></td><td></td><td></td><td>1512</td></tr></table>1</td></tr>
<tr><td>Phase 7:
• Children who are already secure with multiplication for TU × U and TU × TU should have little difficulty in using the same method for HTU × TU.
Again, the carry digits in the partial products are usually carried mentally.</td><td>Phase 6

286 × 29 is approximately 300 × 30 = 9000.

2 8 6
2 9 x
2 5 7 4
5 7 2 0
8 2 9 4</td></tr>
</table>

Mental methods for division of whole numbers

Foundation	Count in tens. Count in twos.
Year 1	Count on and back in tens from and back to zero. Count on in twos from zero. Count in steps of five from zero. Begin to count in steps of three from zero. Begin to know what each digit in a two-digit number represents. Partition a 'teens' number and begin to partition larger two-digit numbers into a multiple of ten and ones.
Year 2	Count in hundreds from and back to zero. Count on in twos from and back to zero. Count in steps of 3, 4 or 5 to at least 30, from and back to zero. Know what each digit in a two-digit number represents, and partition two-digit numbers into a multiple of ten and ones. Understand the operation of division as grouping (repeated subtraction) or as sharing equally. Interpret 8 ¸ 2 as 'how many 2s make 8?' Know by heart multiplication and associated division facts for the 2 and 10 times-tables and derive quickly the corresponding division facts. Use known number facts and place value to carry out mentally, simple divisions. Derive quickly halves of even numbers to 20 and begin to halve of multiples of 10 up to 100.
Year 3	Count on in steps of 3, 4 or 5 from any small number to at least 50 and then back again. Know what each digit represents, and partition three-digit numbers into a multiple of 100, a multiple of ten and ones. Know by heart multiplication and associated division facts for the 2, 5 and 10 timestables. Know that dividing a whole number by 1 leaves the number unchanged. Begin to know the 3 and 4 times-tables. Understand that 16 ¸ 2 does not equal 2 ¸ 16. Understand that division reverses multiplication and solve division calculations by using multiplication strategies. Understand the idea of a remainder. Make sensible decisions about rounding up or down after a division in the context of a problem. To divide by 10, shift the digits one place to the right. Use known number facts and place value to carry out mentally, simple divisions. Say a division statement corresponding to a given multiplication statement.
Year 4	Begin to relate division and fractions. Partition numbers into thousands, hundreds, tens and ones. Multiply and divide any integer up to 1000 by 10 and understand the effect. Begin to divide by 100. Know by heart multiplication and associated division facts for 2, 3, 4, 5 and 10 timestables. Use known facts and place value to multiply and divide integers, including by 10 and then 100. Use known facts and place value to add or subtract a pair of numbers mentally. Use doubling and halving, and factors.
Year 5	Relate division and fractions. Know what each digit represents in a number with up to two decimal places. Multiply and divide any positive integer up to 10 000 by 10 or 100 and understand the effect. Know by heart all multiplication and associated division facts up to 10 x 10. Use known facts and place value to add or subtract a pair of numbers mentally. Use known facts and place value to multiply and divide mentally.
Year 6	Multiply and divide decimals mentally by 10 or 100, and integers by 1000, and explain the effect. Consolidate knowing by heart multiplication and associated division facts up to 10 x 10 Partition (e.g. 87 x 6 = (80 x 6) + (7 x 6)). Use known number facts and place value to consolidate mental division.

Written methods for division of whole numbers

<table>
<tr><td>Phase 1: Recording division as jumps on a number line, leading to jumps on an open number line.</td><td></td></tr>
<tr><td>Phase 2: Mental division using partitioning
• Mental methods for dividing TU ÷ U can be based on partitioning and on the distributive law of division over addition. This allows a multiple of the divisor and the remaining number to be divided separately. The results are then added to find the total quotient.
• Many children can partition and multiply with confidence. But this is not the case for division. One reason for this may be that mental methods of division, stressing the correspondence to mental methods of multiplication, have not in the past been given enough attention.
• Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by 6.</td><td>Phase 2
One way to work out TU ÷ U mentally is to partition TU into a multiple of the divisor plus the remaining units, then divide each part separately.
Informal recording in Year 4 for 84 ÷ 7 might be:
84
70 + 14
↓ ↓ ÷ 7
10 + 2 = 12
In this example, using knowledge of multiples, the 84 is partitioned into 70 (the highest multiple of 7 that is also a multiple of 10 and less than 84) plus 14 and then each part is divided separately using the distributive law.</td></tr>
<tr><td>Phase 3: ‘Expanded’ method for HTU ÷ U
• This method is based on subtracting multiples of the divisor from the number to be divided, the dividend.
• For TU ÷ U there is a link to the mental method.
• As you record the division, ask: ‘How many nines in 90?’ or ‘What is 90 divided by 9?’
• Once they understand and can apply the method, children should be able to move on from TU ÷ U to HTU ÷ U quite quickly as the principles are the same.
• This method, often referred to as ‘chunking’, is based on subtracting multiples of the divisor, or ‘chunks’. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract.
• Chunking is useful for reminding children of the link between division and repeated subtraction.
• However, children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.</td><td>Phase 3
97 ÷ 9
9)97
− 90 9×10
7
Answer: 10 R 7
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6)196
− 60 6×10
136
− 60 6×10
76
− 60 6×10
16
− 12 6× 2
4 32
Answer: 32 R 4</td></tr>
<tr><td>Phase 4
The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. Estimating for HTU ÷ U involves multiplying the divisor by multiples of 10 to find the two multiples that ‘trap’ the HTU dividend.
Estimating has two purposes when doing a division:
~ to help to choose a starting point for the division;
~ to check the answer after the calculation.
Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right.</td><td>To find 196 ÷ 6, we start by multiplying 6 by 10, 20, 30, … to find that 6 × 30 = 180 and 6 × 40 = 240. The multiples of 180 and 240 trap the number 196. This tells us that the answer to 196 ÷ 6 is between 30 and 40.
Start the division by first subtracting 180, leaving 16, and then subtracting the largest possible multiple of 6, which is 12, leaving 4.
6)196
− 180 6×30
16
− 12 6× 2
4 32
Answer: 32 R 4
The quotient 32 (with a remainder of 4) lies between 30 and 40, as predicted.</td></tr>
<tr><td>Phase 5: Long division
The next step is to tackle HTU ÷ TU, which for most children will be in Year 6.
The layout on the right, which links to chunking, is in essence the ‘long division’ method. Recording the build-up to the quotient on the left of the calculation keeps the links with ‘chunking’ and reduces the errors that tend to occur with the positioning of the first digit of the quotient.</td><td>Phase 5
How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560.</td></tr>
</table>

Conventionally the 20, or 2 tens, and the 3 units forming the answer are recorded above the line, as in the second recording.	$24\overline{)\,560}$ $20 - \underline{480}$ $\quad 24\times 20$ $\quad\ 80$ $3 \quad \underline{72}$ $\quad 24\times 3$ $\quad\ 8$ Answer: 23 R 8 In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below. $\begin{array}{r} 23 \\ 24\overline{)\,560} \\ -\underline{480} \\ 80 \\ -\underline{72} \\ 8 \end{array}$ Answer: 23 R 8